Pergamon

www.elsevier.com/locate/jappmathmech

PII: S0021-8928(01)00106-X

J. Appl. Maths Mechs, Vol. 65, No. 6, pp. 1001–1010, 2001

0021-8928/01/$—see front matter

PREDICTION OF THE GROWTH OF FATIGUE CRACKS TAKING ENVIRONMENTAL FACTORS INTO ACCOUNT†

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(*Received* 9 *February* 2001)

A model of corrosion fatigue is proposed which takes into account the main phenomena of a mechanical nature, namely, the transfer of the active agent from the month of the crack to its tip, the accumulation of mechanical damage due to cyclic loads and the breakdown of the stability conditions in the body-with-cracks – load – environment system as the reason for the propagation of the crack tip. Particular attention is devoted to the mechanism by which active agent is transferred. In addition to diffusion, convective transfer as a consequence of the change in the shape and dimensions of the crack cavity is taken into account. The results of modelling are supplemented with diagrams, which relate the rate of growth of the crack with the range of the stress intensity factor, the concentration of the agent at the mouth of the crack and the characteristics of the cycle, equal to the ratio of the extremal values of the applied stresses in each cycle. The results are compared with experimental data. © 2002 Elsevier Science Ltd. All rights reserved.

Corrosion fatigue, i.e. the growth of cracks under the combined action of cyclic loads and an aggressive environment, is the result of the interaction of a number of mechanical, physicochemical and electrochemical processes. The qualitative side of processes of a non-mechanical nature has been investigated in some detail by specialists in the area of metal corrosion and those applied areas for which corrosion fatigue is related to a number of phenomena which have a considerable effect on reliability, safety and longevity of industrial units [1–3]. However, purely mechanical factors also play a decisive role in corrosion fatigue.

Among the mechanical phenomena which accompany the growth of corrosion fatigue cracks and which largely determine their rate of growth, we can distinguish three groups; the transfer of aggressive agent from the mouth of the crack to its tip, the accumulation of mechanical damage in the active zone (around the crack tip) and, possibly, in the far field, since the tip of the crack may progress as far as desired during growth, the balance of forces and energy in the body-with-cracks – load system, on which the stability of the system and, consequently, the possibility of crack growth, depend, and also the extent of its progress. Chemical and electrochemical phenomena are outside the scope of this list. They include direct solution, anode and cathode electrochemical reactions, various forms of embrittlement etc. When analysing corrosion fatigue from the point of view of continuum mechanics, one must consider phenomena of a non-mechanical nature from a purely phenomenological point of view. This approach is even more justified by the fact that in corrosion fatigue mechanical and physicochemical damage occur concurrently, affecting the mechanical properties of the material in the active zone, for example, reducing the value of the specific work of the damage or increasing the yield point.

A theory of the growth of fatigue cracks was proposed in [4], based on a synthesis of fracture mechanics and the mechanics of the accumulation of scattered damage. The growth of macroscopic cracks was regarded as the result of the interaction of the accumulation of microdamage and the stability conditions of the damaged body as a mechanical system. One of the problems consists of determining the generalized forces which occur in the equilibrium and stability conditions of the body-with-cracks – load system. In linear fracture mechanics quantities like the intensity of the release of energy and the J-integral play the role of such generalized forces.

A calculation of the generalized forces in fatigue fracture mechanics involves an isochronous variation of the state of the body-with-cracks load system. In this variation it is necessary to take into account the prehistory of the damage, deformation and accumulation of microdamage and the crack growth. A detailed account of the theory can be found in [5–7]. In the case of corrosion fatigue cracks and corrosion cracking under stress it is necessary to include in the model, damage of non-mechanical origin,

†*Prikl. Mat. Mekh.* Vol. 65, No. 6, pp. 1033–1044, 2001.

supplementing the system of governing relations by equations which describe the process by which corrosion damage builds up.

Attempts to provide a mathematical model of corrosion fatigue and corrosion cracking under stress were first made in [8–11]. In [10] insufficient attention was paid to the transfer of the agent from the mouth of the crack to its tip. More exactly, instead of the transfer equations, phenomenological relations were used, which connect the concentration at the mouth with the concentration at the tip.

The role of mass transfer in the crack cavity changes considerably depending on its depth. For fine cracks (for so-called crevice corrosion) the concentration at the crack tip may be identical with the concentration in the surroundings. For deep cracks, mechanical damage is the predominant factor, so the problem of mass transfer plays a secondary role. From the point of view of this problem, cracks of medium depth, for example, from 1 to 10 mm, are of the greatest interest. In addition to the diffusion mechanism, a purely hydrodynamic mechanism, the role of which increases in the case of cyclic loading, acquires considerable importance. A fatigue crack changes its form and volume within each cycle. In this case some of the agent is removed from the cavity and a new batch of agent enters into it in the next opening. This "pump" effect increases the mass transfer process, making the concentration at the tip of the crack close to its value in the surroundings. On the other hand, agent is absorbed at the tip (partly – and also on the side walls of the crack). Under these conditions an analysis of the effect of mass transfer on the growth of corrosion fatigue cracks becomes of particular interest.

1. MASS TRANSFER TO THE CRACK TIP

Consider a surface crack of mode I under the conditions of the plane problem. The body is loaded "at infinity" by cyclic stresses $\sigma_\infty(t)$ with extremal values in each cycle of $\sigma_\infty^{\max}$ and $\sigma_\infty^{\min}$ and a cycle characteristic $R = \sigma_\infty^{\min}/\sigma_\infty^{\max}$. We will denote the depth of the crack by $a(t)$ and the crack profile by $h(x, t)$. In general, the configuration of the fatigue crack profile may be extremely complex (Fig. 1a) Henceforth, we will use simplified configurations, in particular, those shown in Fig. 1(b–d).

Mass transfer consists of a combination of several processes, such as diffusion, due to the concentration gradient of the aggressive agent, and ion migration, governed by the electrochemical potentials. In this paper we will consider the case when electrochemical effects are negligibly small. The concentration of "fresh" agent, i.e. at the mouth to the crack, will be denoted by c_e, the concentration at the tip will be denoted by c_t and the concentration at $0 < x < a$ will be denoted by $c(x, t)$.

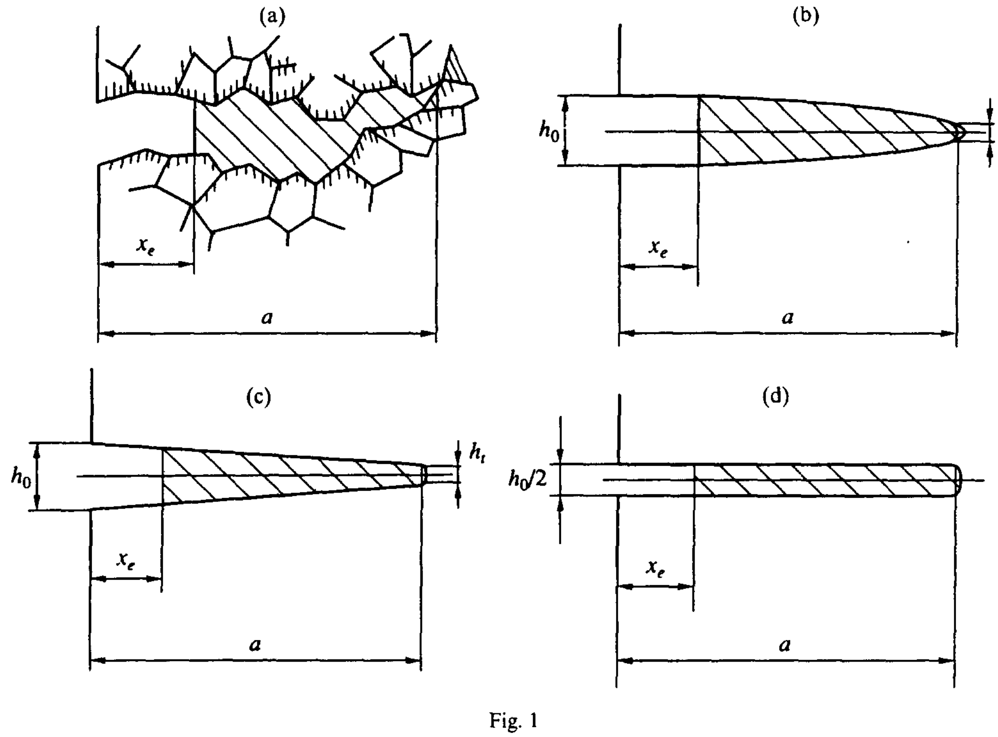

Fig. 1

The crack opening is small compared with it length and hence transfer of agent inside the crack will be described by the equation

$$\frac{\partial(hc)}{\partial t}+v\,\frac{\partial(hc)}{\partial x}=\frac{\partial}{\partial x}\left(hD\frac{\partial c}{\partial x}\right) \tag{1.1}$$

Here $v(x, t)$ is the rate of convective transfer of agent in the cavity in the direction of the x axis and D is the diffusion coefficient. The boundary conditions when first-order reactions occur at the tip take the form

$$c=c_e,\quad x=e_e;\quad D\partial(hc)/\partial x=kh(c_s-c),\quad x=a \tag{1.2}$$

with a reaction (or adsorption) rate coefficient at the crack tip k. Here c_s is the threshold concentration corresponding to a certain equilibrium state or saturation state. In the case of an incompressible fluid and one-dimensional flow, to determine the rate v it is sufficient to use the mass conservation law. We then arrive at the formula

$$v\,(x,t)=\frac{1}{h(x,t)}\frac{\partial}{\partial t}\left(\int\limits_{x}^{x_t(t)} h(\xi,t)d\xi\right) \tag{1.3}$$

Cyclic renewal of the composition of the medium and also reaction of the aggressive agent with the solid phase occurs in the crack cavity. The first of boundary conditions (1.2) contains the coordinate x_e, which corresponds to the transverse cross section of the crack, where the agent can be regarded as "fresh". We define the coordinate x_e as the root of the equation

$$\int\limits_{x_e(t)}^{x_t(t)} h(\xi,t)d\xi=\int\limits_{0}^{x_t(0)} h(\xi,0)d\xi \tag{1.4}$$

the right-hand side of which is equal to the volume of the agent entering the crack cavity before loading starts. Transfer of the boundary condition from the mouth of the crack to the boundary $x = x_e$ enables us to reduce the volume of calculation somewhat when calculating the mass transfer.

The problem of choosing the profile of the edge of the crack has been discussed in the literature mainly in the context of the relation between the aperture of the crack at the tip and at the mouth (12). Equations (1.1)–(1.4) contain the crack profile function $h(x, t)$. Actual cracks have bends and branches (Fig. 1a) and are far from these idealizations, which are used in fracture mechanics. To analyse corrosion fatigue we need to give a schematic description of the form of the cavity occupied by the crack. In this paper we consider elliptical and trapezoidal profiles and a profile that is constant along the length of the opening, as shown in Figs 1(b), (c) and (d), respectively. To determine the opening at the mouth of the crack and at its tip we will use the expressions

$$h_0=\frac{4Z_0\sigma_\infty a(1-\nu^2)}{E},\quad h_t=\frac{Z_t\sigma_\infty^2 a}{E\sigma_Y} \tag{1.5}$$

where σ_Y is the yield stress, while the coefficients Z_0 and Z_t are of the order of unity. The formula for h_0 corresponds approximately to linear fracture mechanics and the formula for h_t gives the opening at the tip of the crack in the framework of the thin plastic zone model. This representation is justified for fairly deep cracks.

We will investigate mass transfer of agent in the crack cavity by computational experiment. Since the growth of corrosion fatigue cracks is accompanied by the interaction of many processes, it is best to consider the mass transfer on the assumption that the crack growth process is specified. More exactly, we will specify the number of cycles N_* before the crack starts to grow, i.e. the duration of the incubation stage, and the rate of advance of the tip da/dN when $N > N_*$.

The change in the concentration of the agent at the tip of the crack is shown in Fig. 2 for different shapes of the crack cavity for the following values of the parameters: $D = 10^{-8}$ m^2/s, $k = 10^{-6}$ m/s and $c_s/c_d = 0.1$. Curve 1 corresponds to an elliptical shape, curve 2 corresponds to a trapezoidal cavity profile and curve 3 corresponds to a crack with a constant aperture along its length (Fig. 1). Although the crack tip does not move, the concentration at the tip decreases due to absorption on the crack surfaces. The concentration subsequently remains almost constant.

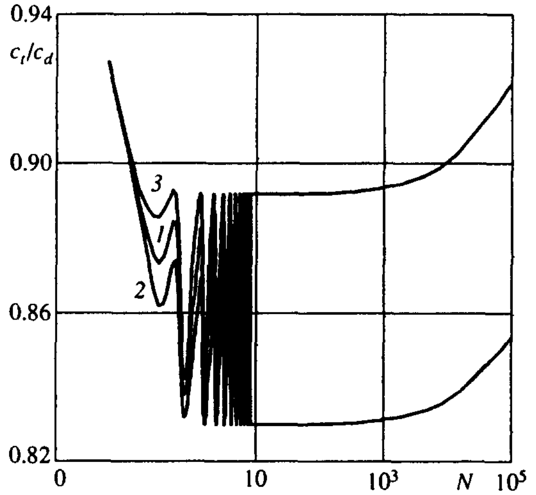

Fig. 2

It should be noted that the change of concentration in a cycle is only shown at the initial stage, containing only 10 loading cycles. Henceforth, the behaviour will be represented by the envelopes, which show the maximum and minimum values of the agent concentration. So as not to crowd the figure, the envelopes are shown for one of the profiles; the behaviour of the envelopes in the remaining cases is easily established by analogy. Adsorption of the agent occurs most intensively if the crack has a profile of constant aperture along the length (curve 3). This profile enables the agent to diffuse to the crack tip more freely, which is a natural result.

Neglect of the convective term in Eq. (1.1) does not lead to any appreciable change in the concentration of the agent at the tip either at the incubation stage or at the crack growth stage. The "pump" effect has a much greater influence. In Fig. 3(b) we show the change in the boundary of "fresh" agent for an elliptical cavity for values of the cycle asymmetry coefficient R = 0.25, 0.5 and 0.75. The greater the amplitude of the stresses the more intensively penetration of the aggressive agent into the crack cavity occurs (Fig. 3a). The fresh agent is displaced from the crack tip when $\sigma_\infty = \sigma_{\infty,\,\min}$ and fills almost the whole of the crack cavity when $\sigma_\infty = \sigma_{\infty,\,\max}$. Hence the effect of closing of the crack during mass transfer of the medium in its cavity is taken into account. The boundary of the "fresh" agent at the crack growth stage approaches the crack tip, intensifying the reaction of newly arriving agent with the material. The nature of the change in the concentration and the coordinates of the boundary of fresh agent are shown in detail only at the, initial stage, which contains only 10 loading cycles. Henceforth the process is represented by the envelopes.

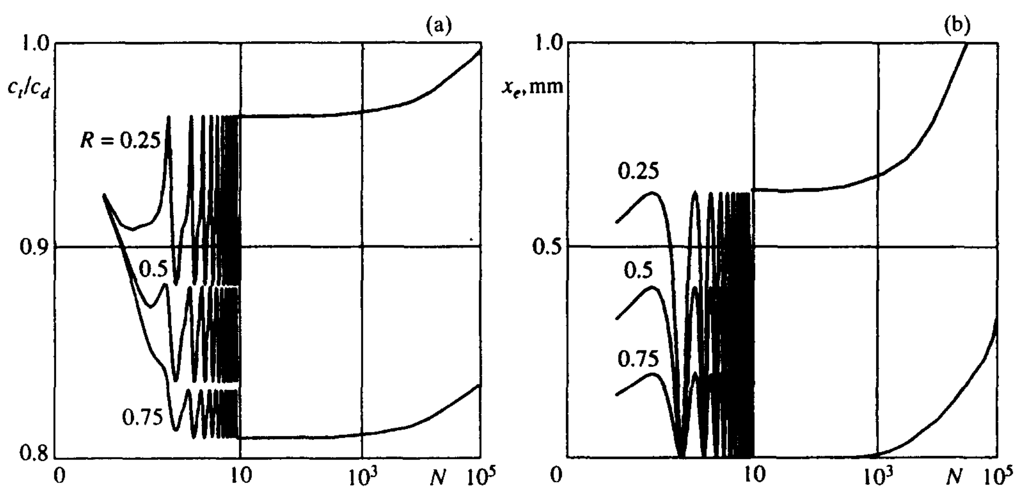

Fig. 3

2. THE ACCUMULATION OF DAMAGE

We will use the simplest (scalar) model, first introduced by Kachanov and Rabotnov (13), to model the processes by which damage accumulates. They proposed to describe scattered mechanical damage using the scalar field $\omega(x, t)$ where $0 \leqslant \omega(x, t) \leqslant 1$. The lower boundary corresponds to the undamaged material and the upper boundary corresponds to the completely damaged material. Since then many generalizations have been proposed, including second, fourth- and higher-order tensors [14]. Nevertheless, the scalar model retains the advantages of considerable simplicity and clarity, and it is also possible to estimate the parameters of the model from a macroscopic experiment. This is particularly important when modelling corrosion fatigue, where it is necessary to distinguish different types of damage. Even when modelling purely mechanical fatigue it is necessary to distinguish between damage ω_f and ω_s from cyclic and slowly changing components of the loading, respectively. In the case of corrosion fatigue it is necessary to introduce additional measures, which distinguish between anode and cathode corrosion damage, hydrogen embrittlement etc. Without making this distinction, we will describe the level of corrosion damage by a scalar measure ω_c. In general we will use a set of scalar fields $\omega_1(x, t), \ldots, \omega_\nu(x, t)$ [7].

We will assume that the damage accumulation is described by first-order differential equations. For a crack with a normal break (mode I) we will use an equation of the threshold-power form [4, 5, 7]

$$\frac{\partial \omega_f}{\partial N} = \left(\frac{\Delta\sigma - \Delta\sigma_{\text{th}}}{\sigma_f}\right)^{m_f}, \quad \frac{\partial \omega_s}{\partial t} = \frac{1}{t_c}\left(\frac{\sigma - \sigma_{\text{th}}}{\sigma_s}\right)^{m_s}, \quad \frac{\partial \omega_c}{\partial t} = \frac{1}{t_c}\left(\frac{c - c_{\text{th}}}{c_d}\right)^{m_c} \tag{2.1}$$

Here $\Delta\sigma$ is the range of breaking stresses at the tip of the crack and along its length, and σ is the average or slowly varying component of this stress. The material parameters σ_f, σ_s and c_d characterize the resistance of the material to damage from cyclic, slowly varying loads and an aggressive medium. We will denote the threshold values of the strengths by $\Delta\sigma_{\text{th}}$, σ_{th} and c_{th} (when $\Delta\sigma < \Delta\sigma_{\text{th}}$, $\sigma < \sigma_{\text{th}}$ and $c < c_{\text{th}}$ the right-hand sides of the corresponding equations should be equated to zero). The exponents m_f, m_s and m_c are closely connected with the exponents of the fatigue curves of the growth of fatigue cracks and, under certain conditions [7], the exponents have close values. The last two equations of (2.1) contain a time constant t_c which can be chosen from considerations of convenience.

The stresses $\Delta\sigma$ and σ depend on the conditions at the tip of the crack, which are characterized using the effective radius of curvature ρ. We will describe the change in this radius with time for a plane mode I crack with depth a by the equation

$$\frac{d\rho}{dt} = \frac{\rho_s - \rho}{\lambda_a}\frac{da}{dt} + (\rho_b - \rho)\frac{d(\psi_f + \psi_s)}{dt} + (\rho_c - \rho)\frac{d\psi_c}{dt} \tag{2.2}$$

This equation takes into account the sharpening of the tip to a value of ρ_s when there is accelerated growth of the crack and its blunting to a value of ρ_b and/or ρ_s when the crack growth slows down. We have used the notation ψ_f, ψ_s and ψ_c for the values of the measures ω_f, ω_s and ω_c on the tip $x = x_t$, and also the notation λ_a for the parameter, which has the dimensions of length.

The use of relatively simple equations (2.1) and (2.2) encounters serious difficulties. Thus the range of stresses $\Delta\sigma$ and the average stress σ depend on the damage level. Treating the measures ω_f and ω_s as additive and the sum $\omega = \omega_f + \omega_s$ as a measure of the cracking, it is natural to introduce into Eqs (2.1) the reduced stresses $\Delta\sigma/(1 - \omega)$ and $\sigma/(1 - \omega)$. Another method of taking the effect of damage into account is to postulate that the material parameters σ_f, $\Delta\sigma_{\text{th}}$, etc. depend on the measures of damage. One other method consists of replacing the right-hand sides in Eqs (2.1) by more complex expressions containing measures of damage. For example, the first equation of (2.1) can be replaced by the following ($n_c > 0, n_f > 0$).

$$\frac{\partial \omega_f}{\partial N} = \left(\frac{\Delta\sigma - \Delta\sigma_{\text{th}}}{\sigma_f}\right)^{m_f} \frac{(1 - \omega_c)^{n_c}}{(1 - \omega_f - \omega_s)^{n_f}} \tag{2.3}$$

In order to calculate the measures of damage we need to know the stress field in the vicinity of the crack tip. In general, this requires the use of numerical methods. However, there is a simple approach based on an analogy between the stress concentration factors $\varkappa$ and the stress intensity factor K. For a plane mode I crack this analogy gives an approximate formula for the stress concentration factor at the tip and the stress distribution in front of the tip

$$\varkappa = 1 + 2Y\left(\frac{a}{\rho}\right)^{1/2}, \quad \sigma = \varkappa\sigma_\infty\left[1 + \frac{4(x-a)}{\rho}\right]^{-1} \tag{2.4}$$

These formulae generalize the well-known Neuber formula; the form factor Y is taken from the corresponding formula for the stress intensity factor. For an edge crack we can put $Y = 1.12$.

To supplement Eqs (2.1)–(2.3) we need to have an equation which describes the kinetics of the change in the thickness λ of the corrosion film. These kinetics are largely similar to the kinetics of the change in the radius ρ

$$\frac{d\lambda}{dt} = \frac{\lambda_s - \lambda}{\lambda_a}\frac{da}{dt} + (\lambda_b - \lambda)\frac{d(\psi_f + \psi_s)}{dt} + (\lambda_c - \lambda)\frac{d\psi_c}{dt} \tag{2.5}$$

Here λ_s, λ_b and λ_c are characteristic dimensions, where the dimension λ_a is analogous to the parameter occurring in Eq. (2.2). When $\lambda_s < \lambda_b \leqslant \lambda_c$ the parameter λ_c can be interpreted as the maximum thickness of the corrosion film. All these parameters are small compared with the length of the crack a. As a rule they are also small compared with the dimension λ_p of the zone in which intensive accumulation of mechanical damage occurs. This enables us to assume a linear distribution of the concentration $c(x, t)$ in the limits of the thickness of the corrosion film $a \leqslant x \leqslant a + \lambda$, which in turn enables us to confine ourselves to considering the concentration c_t at the crack tip and the corresponding measure of damage ψ_c [11].

The effect of damage on the macroscopic parameters of the material is also extremely important. In this model their effect on the value of the crack resistance indicators is particularly important. We will take as the fundamental indicator the value of the specific work of fracture γ. Since insufficient experimental data are available we will assume that the damage measures are additive. We can then assume that

$$\gamma = \gamma_0[1 - \chi(\omega_f + \omega_s + \omega_c)^\alpha] \tag{2.6}$$

where γ_0 is the specific work of damage for the undamaged material, $\alpha \geqslant 1$ and $0 < \chi \leqslant 1$. When $\omega_f + \omega_s + \omega_c = 1$ formula (2.6) gives a value of the residual work of fracture for the cavity of the damaged material. Note that formula (2.6) does not assume superposition of the damages. These measures are connected with one another in an extremely complex way, in particular by Eqs (2.1)–(2.4).

3. A THEORY OF CRACK GROWTH

To describe the growth of corrosion fatigue cracks we will use the theory in [4] and supplement its relations with equations which describe the mass transfer and damage accumulation processes. Consider the body-with-cracks load environment system. We will treat it as a mechanical system with unilateral constraints, which takes into account the irreversibility of cracks in ordinary structural materials. The stability of this system to small changes in the dimensions of the cracks will be investigated using the idea of Griffith variation [4]. This is an isochronous transition to mixed states of equilibrium in which only the crack parameters are subject to variation, and all the equations of deformation equilibrium and compatibility and all the boundary conditions are satisfied, apart from the conditions on the crack tips. The crack parameters $a_1, \ldots a_m$ have the meaning of generalized coordinates, and, in view of the irreversibility of the cracks, their variations satisfy the conditions $\delta a_j \geqslant 0$ $(j = 1, \ldots, m)$.

We will call the state of the system a subequilibrium state if the virtual work of all the external and internal forces (calculated by Griffith variation) $\delta W < 0$. We will call the state an equilibrium state if variations exist on which $\delta W = 0$, and for the remaining variations $\delta W < 0$, and a non-equilibrium state if variations exist on which $\delta W > 0$. Subequilibrium states are stable, non-equilibrium states are unstable, while the stability of equilibrium states depends on the sign of $\delta(\delta W)$, where the second variation is calculated in the Griffith manner [4, 5, 7]. The equilibrium state of the body-with-cracks load system is stable if, for all variations, $\delta(\delta W) < 0$, and unstable if variations exist for which $\delta(\delta W) > 0$.

We will represent the virtual work in the form $\delta W = \delta W_e + \delta W_i + \delta W_f$, where δW_e is the work of external forces, δW_i is the work of internal forces and δW_f is the work done in advancing the tip of the crack. Using the relations

$$\delta W_e + \delta W_i = \sum_{j=1}^{m} G_j \delta a_j, \quad \delta W_f = -\sum_{j=1}^{m} \Gamma_j \delta a_j \tag{3.1}$$

we introduce two groups of generalized forces. Generalized forces G_j will be called active (forces which advance cracks) and generalized forces Γ_j will be called passive (resistance forces). A crack does not grow if all $G_j < \Gamma_j$. If the equality $G_k = \Gamma_k$ is attained for one of a_k the crack may begin to grow. This growth will be stable if $\partial G_k/\partial a_k < \partial \Gamma_k/\partial a_k$ and unstable if $\partial G_k/\partial a_k > \partial \Gamma_k/\partial a_k$. If at least one of the relations between the generalized forces takes the form $G_k > \Gamma_k$ the system becomes unstable with respect to the corresponding generalized coordinate. This indicates either final fracture of the body or a jump to the next subequilibrium (stable) state.

The relations between the generalized forces change when the damage accumulates. A typical situation is where, at the start of loading, the system is in a subequilibrium state. The number of cycles N_* for which the inequality $G < \Gamma$ first breaks down corresponds to the beginning of crack growth. This growth can be both continuous and abrupt. The crack grows continuously if the mechanism by which the damage accumulates continuously satisfies the condition $G = \Gamma$, $\partial G/\partial a < \partial \Gamma/\partial a$. Abrupt growth is observed when the system transfers from one subequilibrium state to another. This occurs either due to overload, when the inequality $G > \Gamma$ is satisfied, or due to gradual accumulation of damage. In the latter case a jump occurs when an unstable equilibrium state is reached, i.e. when the condition $G = \Gamma$, $\partial G/\partial a > \partial \Gamma/\partial a$ is reached [7].

In the case of fatigue cracks the generalized forces depend on the size of the crack, the loading parameters and the measures of mechanical damage. For corrosion fatigue they also depend on the measure of corrosion damage $\omega_c(t)$ and the thickness of the corrosion film $\lambda(t)$.

4. NUMERICAL EXAMPLES

The computational procedure includes solving the problem of the mass transfer of agent from the mouth of the crack to its tip, calculation of the stress field at the tip and along its length, solution of the differential equations describing the damage accumulation, calculation of the characteristics of the material, taking the accumulated damage into account, calculation of the active generalized forces and the generalized resistance forces, and a check of the stability of the system with respect to small increments in the crack length. If the crack advances by a small step, the whole procedure is repeated for the new crack configuration. One of the features of the calculations is the use of two time scales. In order to describe the motion of the agent in the crack cavity when there is a cyclic change in its volume, it is necessary to use "fast" time, splitting the cycle duration into 10 or more steps. The damage accumulation and the crack growth can be described by treating the number of cycles N as a continuous parameter. In particular, the relations between the generalized forces can be considered using the maximum difference between the generalized forces in the limits of the cycle $t_{N-1} < t \leq t_N$:

$$H(N) = \max_{t_{N-1} < t \leq t_N} \{G[a(t), \sigma_\infty(t), \psi(t)] - \Gamma[a(t), \sigma_\infty(t), \psi(t)]\} \tag{4.1}$$

Here $\psi(t)$ is the set of all parameters characterizing the level of damage at the tip, the effective radius of curvature and the thickness of the corrosion film. The crack does not grow when $H(N) < 0$ and grows stable when $H(N) = 0$, $\partial H(N)/\partial a < 0$. In order to avoid the use of two time scales and excessive costs in computer time, beginning with a certain N it is best to solve the mass-transfer problem in a "slow" time scale. In this case the first of boundary conditions (1.2) is formulated for a value of x_e corresponding to the mean position of the boundary of "fresh" agent within a cycle.

It was shown in [15, 16] that the effect of damage on the value of the active generalized force can be neglected. The dimensions of the plastic zone will be assumed small compared with the depth of the crack, and the material will be assumed to be linearly elastic. As it applies to a mode I crack this means that Irwin's formula is applicable, namely,

$$G = \frac{K^2(1-\nu^2)}{E} \tag{4.2}$$

where $K = Y\sigma_\infty(\pi a)^{1/2}$ is the stress intensity factor for the edge of the crack with form factor $Y = 1.12$, E is Young's modulus and ν is Poisson's ratio. For the generalized resistance force we use expression (2.6)

$$\Gamma = \gamma_0[1 - \chi(\psi_f + \psi_s + \psi_c)^\alpha] \tag{4.3}$$

where ψ_f, ψ_s and ψ_c are the values of the corresponding measures of damage at the crack tip.

Calculations were carried out for the following data: E = 200 GPa, ν = 0.3, γ_0 = 10 kJ/m^2 and $\alpha = \chi = 1$. The parameters in the damage accumulations equations (2.1) were taken as follows: $\sigma_f = \sigma_s$ = 5 GPa, $\Delta\sigma_{th}$ = 125 MPa, σ_{th} = 250 MPa, and $m_f = m_s = m_c = 4$. The parameter c_d is used for normalization, i.e. all values of the concentration are expressed in terms of the ratio c/c_d. To describe the mass transfer we took the numerical data used above. In Eq. (2.2) we assumed that ρ_s = 10 μm and $\rho_b = \lambda_a$ = 100 μm. Most of the quantities used were not estimated directly by experiment; their values were chosen in such a way that the final results were in agreement with published experimental data [2, 16, 17].

The concentration at the mouth of the crack will be specified by the ratio c_e/c_d. This ratio, and also the extremal value of the applied stresses, will be assumed constant during the whole process of the production and growth of the crack. We will assume that the stress σ_∞ can be expressed in terms of its range $\Delta\sigma_\infty$ and the cycle asymmetry factor R. All the graphs presented below were obtained for $\Delta\sigma_\infty$ = 150 MPa and R = 0.5. The loading frequency was taken to be $f = 10^{-2}$ Hz. Figure 6 (see below) is an exception; it was constructed for different values of the frequency.

The growth of a corrosion fatigue crack for different concentrations of the active agent is shown in Fig. 4. Curves 1–5 are drawn for dimensionless parameters c_e/c_d = 0, 0.25, 0.5, 0.75 and 1, respectively, i.e. beginning from the case of a neutral medium to a medium of high concentration. However, one can interpret these curves as referring to agents of different chemical composition. The change in the depth of the crack as a function of the number of cycles is shown in Fig. 4(a). A graph similar to the generally accepted curve of the growth of fatigue cracks is shown in Fig. 4(b), where the rate of growth da/dN is represented as a function of the range of the intensity factor $\Delta K = Y\Delta\sigma_\infty(\pi a)^{1/2}$. Some irregularity in the form of the non-monotonic dependence of the rate on the number of cycles is observed on the initial parts of the graphs. Hence on these parts we must carry out a numerical integration with a short time step. The irregularity later disappears, which enables us to change to integration in blocks. To economise on computer time the boundary condition for the diffusion equation is transferred to the boundary of the "fresh" agent in the middle of the loading cycle. For a high concentration of agent the growth of short cracks is mainly controlled by corrosion damage. When the crack becomes fairly deep, mechanical damage becomes decisive. This can be seen in Fig. 4(b), where the curves corresponding to different c_e/c_d show a tendency to converge as ΔK increases. The slope of the middle part of the curves (i.e. the analogue of the Paris factor for the usual diagrams of the growth of fatigue cracks) is close to the value m = 4. We recall, that in this numerical example $m_s = m_f = m_c = 4$.

The relation between the measures of damage, or, more exactly, between their contribution to the overall measure $\psi = \psi_s + \psi_f + \psi_c$, depends on the level of damage and the concentration of the agent. The growth of a corrosion fatigue crack includes, generally speaking, a component which describes corrosion cracking, the contribution of which depends very much on the frequency. The ratio between the individual components varies as the crack grows, as illustrated in Fig. 5, where we show three

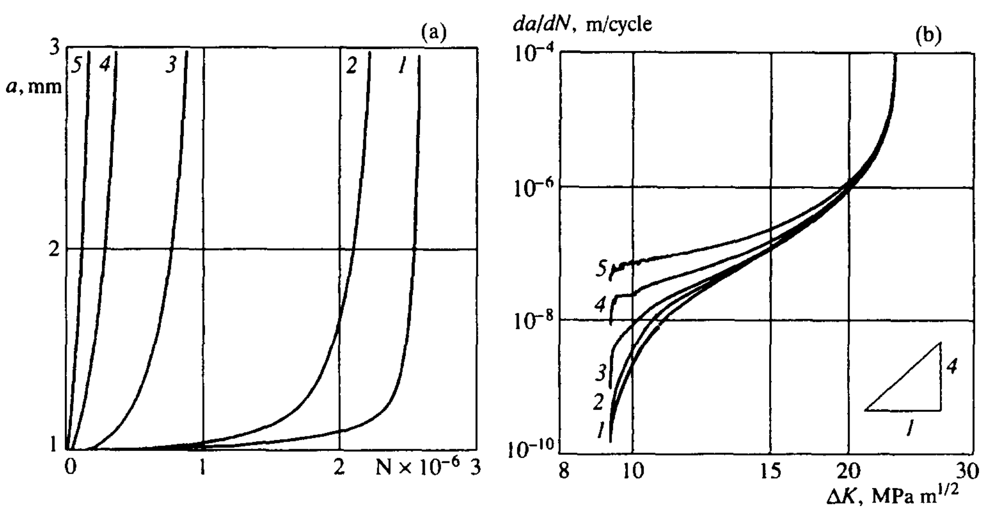

Fig. 4

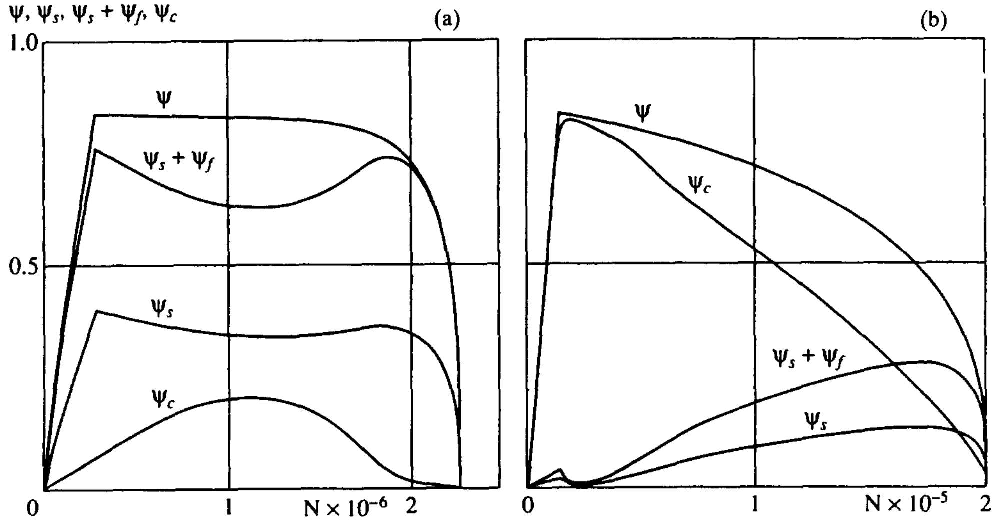

Fig. 5

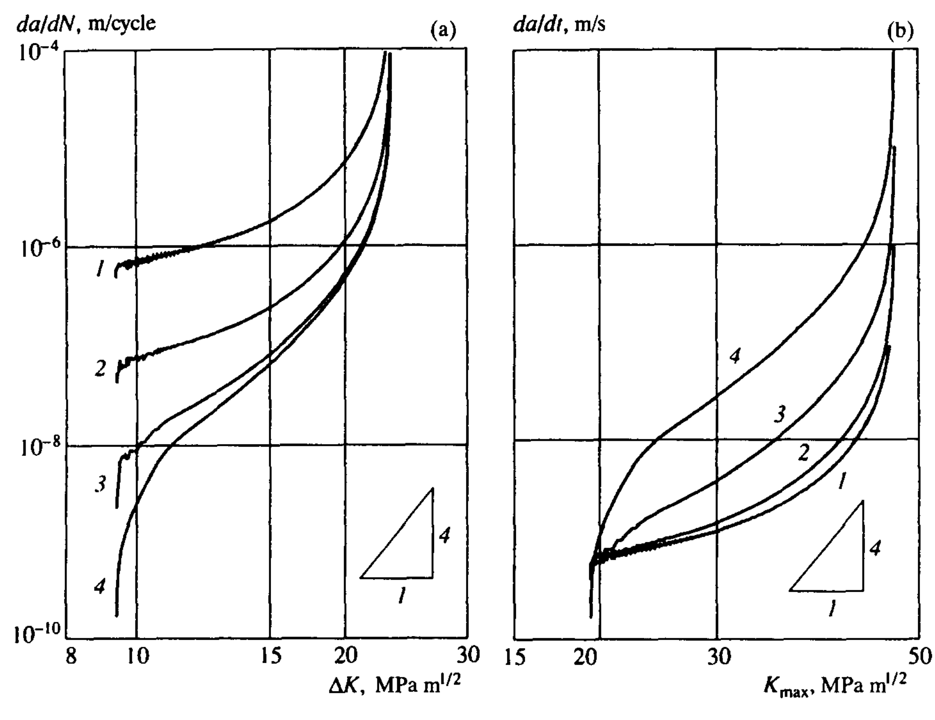

Fig. 6

measures of damage, one of which corresponds to the contribution of constant stresses. Figure 5(a) was drawn for $c_e/c_d = 0.25$ and Fig. 5(b) for $c_e/c_d = 1$. The overall measure of damage increases while the crack tip is motionless, and begins to decrease as the crack tip advances, reaching small values at the instant of final fracture. The contribution of each type of damage depends on the agent concentration, the ratio between the duration of the cycle and the characteristic mass transfer time and on the range of applied stresses and on the cycle asymmetry factor. In the example considered, the contribution of the damage from the slowly varying part of the stresses is comparable with the contribution of the cyclic stresses. The corrosion component of the stress increases, at an early stage and decreases as the crack grows.

The loading frequency is also an important factor. Corrosion develops in physical time, while mechanical damage increases as the number of cycles increases. Measuring the crack growth rate as da/dN naturally represents it as a function of ΔK. If the crack growth rate is measured as da/dt, it is natural to consider it as a function of the maximum value of the intensity factor within a cycle. Figure 6 shows two types of diagrams, constructed for $\Delta\sigma_\infty = 150$ MPa, $R = 0.5$, $c_e/c_d = 1$ and loading frequencies $f = 10^{-3}, 10^{-2}, 10^{-1}$ and 1 Hz (curves 1–4, respectively). In Fig. 6 the crack growth rate is represented as a function of ΔK (*a*) and as a function of $K_{\max} = \Delta K(1 - R)^{-1}$ (b).

The effect of the frequency is considerable. When the frequency f changes from 10^{-3} to 1 Hz the difference in the rates may reach two orders or more. The way the curves diverge depends on the method used to measure the rate. In Fig. 6(a) the crack growth rate is specified as da/dN while in Fig. 6(b) it is specified as da/dt. At low loading frequencies there is a tendency for a plateau to form on the growth diagrams in the graphs of da/dN against ΔK. This occurs due to the predominance of the corrosion mechanism in the early stage. Final fracture occurs close to values of $\Delta K = K_c(1 - R)$ or $K_{\max} = K_c$, where K_c is the crack resistance characteristic for the undamaged material.

The proposed model is phenomenological, particularly in the part relating to physical and physico-chemical processes. Many parameters of the model are not amenable to a direct experimental estimate, although, in principle, they can be determined from the data of a macroscopic experiment. The final results, represented in the form of diagrams of fatigue crack growth, are in qualitative agreement with existing experimental data [17, 18]. With a successful choice of the "free" parameters of the model one can obtain quantitative agreement. However, the purpose of this paper is not to replace tests on corrosion fatigue (which, anyway, are of a relatively routine nature), but to clarify the mechanisms by which the various processes which accompany corrosion fatigue crack growth interact.

This research was partially supported by the Russian Foundation for Basic Research (99-01-00282, 00-15-96138 and 01-01-06105).

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Translated by R C.G.